Modelling influence propagation in social networks

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theoretical foundations
of machine learning, Będlewo

Introduction

- influence propagation in social networks
- applications - propagation of ideas & behaviours, adoption of technology, **viral marketing**
- KKT - influence maximisation problem

D. Kempe, J. Kleinberg, E. Tardos. *Maximizing the spread of influence through a social network*. KDD '03. ACM, 2003

D. Kempe, J. Kleinberg, E. Tardos. *Influential nodes in a diffusion model for social networks*. ICALP'05. Springer-Verlag, 2005

G - network of individuals (directed graph)

$\Downarrow$

S_0 - initial set of k individuals (k is a parameter)

$\Downarrow$

random propagation

$\Downarrow$

$\varphi(S_0)$ - final set of influenced (also: active) individuals

Influence Maximisation Problem:

find S_0 which maximises

(the expected value of)

the cardinality of $\varphi(S_0)$

$(\Omega, 2^{\Omega}, \mathbb{P})$ - a fixed probability space, Ω is finite.

Social network: a directed graph $G = (V, E)$

- vertices $V = \{v_1, \dots, v_N\}$
- edges $E \subset (V \times V) \setminus \{(v, v) \mid v \in V\}$

Propagation: A family of stochastic processes $\mathcal{P} := \{\mathcal{P}_S\}_{S \in 2^V}$, where $\mathcal{P}_S : \Omega \times \mathbb{N} \ni (\omega, i) \mapsto S_i(\omega) \in 2^V$ such that

1. $S_0(\omega) := S$
2. $v \in S_i(\omega) \setminus S_{i-1}(\omega) \Rightarrow$ there exists $u \in S_{i-1}(\omega)$ and $(u, v) \in E$

Influence function: expected final number of active vertices when starting from a given set

$$\sigma\colon 2^V \to \mathbb{N},\ \sigma(S) = \mathbb{E}(\#S_T), \qquad S_T \text{ - final set}$$

Influence maximisation problem with parameter $k \in \mathbb{N}$:
to find S^* such that $\sigma(S^*) = \max\{\sigma(S) \mid S \subset V,\ \#S = k\}$

the influence function $\sigma\colon 2^V \to \mathbb{N}$ is called

- *monotone* if $\sigma(S) \leq \sigma(\bar{S})$
- *submodular* if $\sigma(S \cup \{v\}) - \sigma(S) \geq \sigma(\bar{S} \cup \{v\}) - \sigma(\bar{S})$

for all sets $S \subset \bar{S}$

Theorem (Nemhauser, Wolsey, Fisher (1978); KKT (2003))

If $\sigma : 2^V \to \mathbb{N}$ is monotone and submodular then the set of vertices S chosen by the greedy algorithm satisfies $\sigma(S) \geq (1 - 1/e)\sigma(S^*)$, where S^* is the solution to the influence maximisation problem with parameter k. $((1 - 1/e) \approx 63\%)$

By the *greedy algorithm* we mean:

1. $S := \emptyset$
2. for i= 1 to k

. $\quad v_i \leftarrow \mathtt{argmax}_{v \in V \setminus S} (\sigma(S \cup \{v\}) - \sigma(S))$

. $\quad S \leftarrow S \cup \{v_i\}$

Independent Cascade Model

For $(u, v) \in E$ we have $p_{(u,v)} \in (0, 1]$ and a random variable

$$X_{(u,v)} \colon \Omega \to \{0, 1\} \quad (\textit{was the activation attempt successful?})$$

$$\mathbb{P}(X_{(u,v)} = 1) = \begin{cases} p_{(u,v)} & (u, v) \in E \\ 0, & (u, v) \notin E \end{cases}$$

These random variable are assumed to be **independent**.

$$S_i(\omega) := S_{i-1}(\omega) \cup A_i(\omega)$$
$$A_0(\omega) := S$$
$$A_{i+1}(\omega) := \{v \in V \setminus S_i(\omega) \ \mid \ \exists_{u \in V} \ X_{(u,v)}(\omega) = 1 \ \wedge \ u \in A_i(\omega)\}$$

Theorem (KKT'03)

The influence maximisation problem in ICM is NP-hard.

Theorem (KKT'03)

The influence function in ICM is monotone and submodular.

Generalised Cascade Model (GCM)

$X_v := (X_{(u_i,v)} \mid u_i \in V,\ i = 1, \dots, N)$ are independent

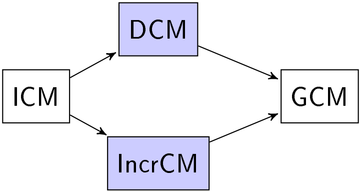
Parameters: $p_v(u, S) := \mathbb{P}(X_{(u,v)} = 1 \mid \forall_{w \in S}\ X_{(w,v)} = 0)$

Increasing Cascade Model (IncrCM).

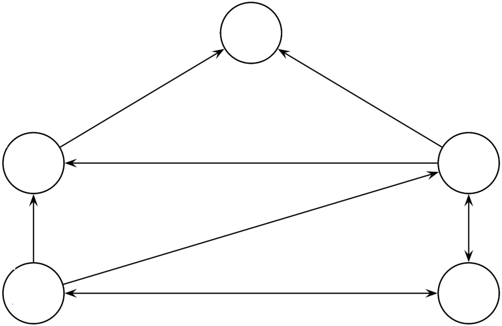
$$S \subset \bar{S} \implies p_v(u, S) \leq p_v(u, \bar{S})$$

Decreasing Cascade Model (DCM) /appeared already in KKT/

$$S \subset \bar{S} \implies p_v(u, S) \geq p_v(u, \bar{S})$$



Threshold models



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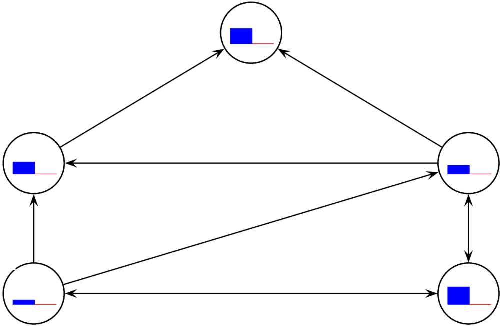

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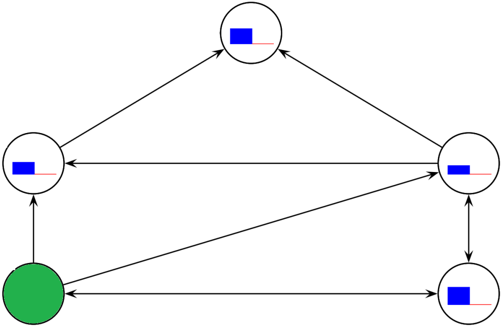

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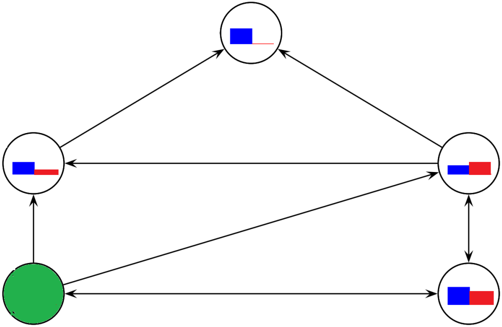

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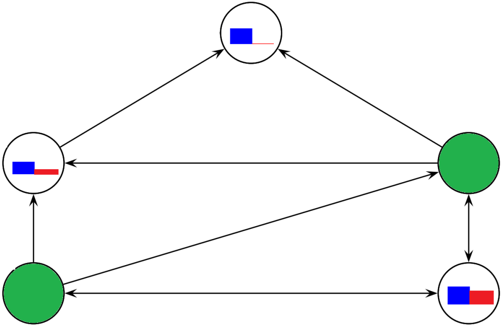

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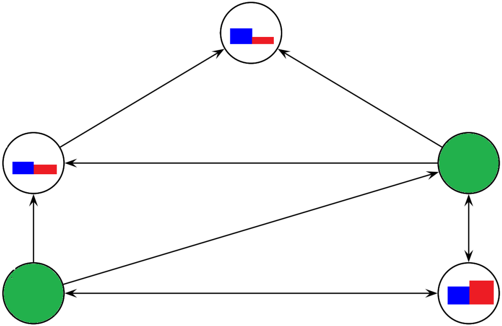

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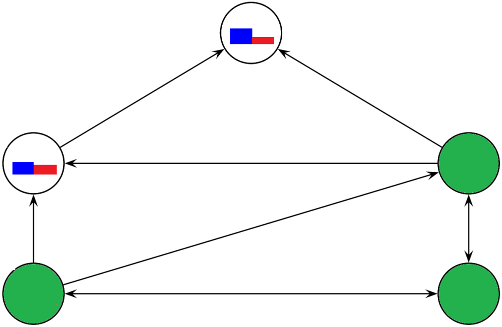

Linear Threshold Model

For every edge in $G = (V, E)$ we have $b_{(u,v)} \in (0, 1]$ such that for all $v \in V$

$$\sum_{u:\ (u,v) \in E} b_{(u,v)} \leq 1 \quad \text{(normalisation)}$$

Thresholds: $\theta_v : \Omega \to [0, 1]$, independent, uniformly distributed
Active sets:

$$S_i(\omega) := S \cup \{v \in V \mid \sum_{u \in S_{i-1}(\omega)} b_{(u,v)} \geq \theta_v(\omega)\}$$

linear accumulation of influence

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Non-normalised Linear Threshold Model

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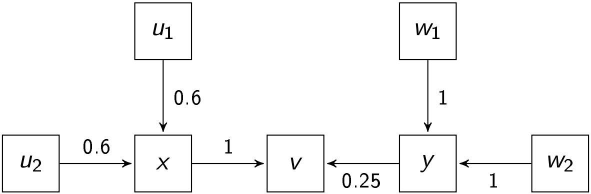


Generalised Threshold Model

We consider for each v the ***activation function***

$$f_v : 2^{N_v} \to (0,1]$$

which is assumed to be monotone and to satisfy $f_v(\emptyset) = 0$.

Active sets:

$$S_i(\omega) := S \cup \{v \in V \mid f_v(S_{i-1}(\omega)) \geq \theta_v(\omega)\}$$

In LTM: $f_v(S) = \sum_{u \in S} b_{(u,v)}$

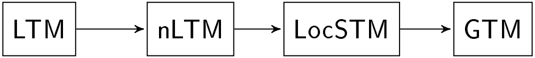
In nLTM: $f_v(S) = \min\left(\sum_{u \in S} b_{(u,v)}, 1\right)$

Locally Submodular Threshold Model

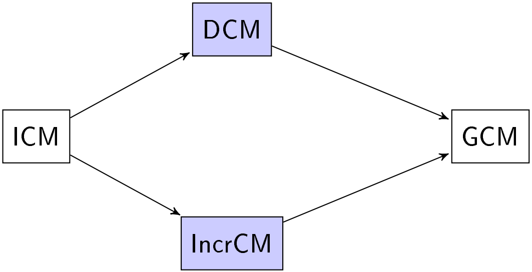
All the activation functions f_v are required to be submodular.

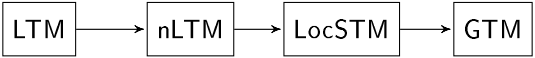
The submodularity of σ in this model was conjectured by KKT (2003) and proved by Mossel & Roch (2010).

In nLTM we have $f_v(S) = \min(1, \sum_{u \in S} b_{u,v})$ - submodular!



Generalised Models: GCM & GTM (KKT)





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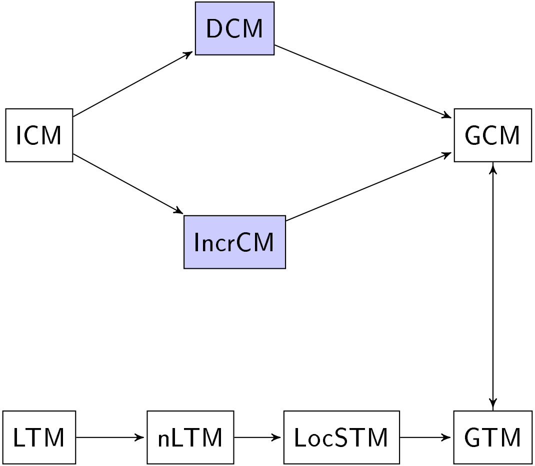

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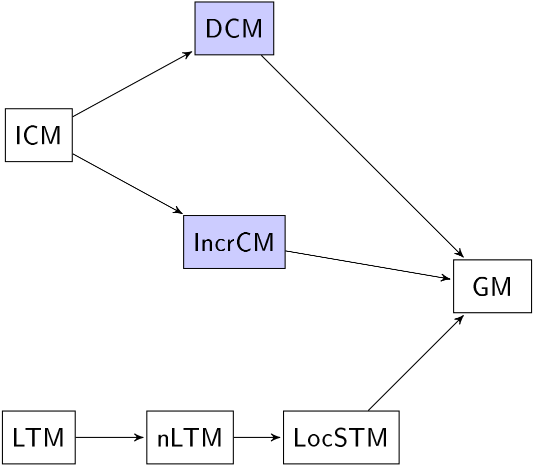


Results about DCM & IncrCM

DCM the influence function is submodular (delayed propagation processess in KKT'05)

DCM it is a special case of LocSTM

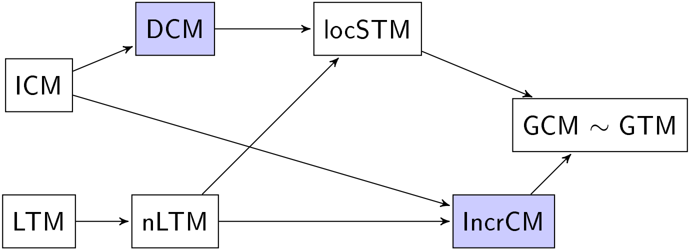
IncrCM the influence function is not submodular

IncrCM it is not a special case of LocSTM

necessary condition for LocSTM:

$$f_v(S) \leq \sum_{u \in S} f_v(u)$$

IncrCM nLTM is a special case of IncrCM & it is submodular



The formalised framework for models of influence propagation:

- unifies cascade and threshold models & clarifies their structure
- reveals new connections & leads to new models